

3. ESERCIZI

EQUAZIONI CON MODULO

LIVELLO BASE

1. Completa:

$$|-12| = \dots\dots\dots \quad |6| = \dots\dots\dots \quad |\sqrt[3]{-27}| = \dots\dots\dots \quad |-5 + \sqrt{2}| = \dots\dots\dots$$

$$\left| \frac{1}{2} - \frac{\pi}{4} \right| = \dots\dots\dots \quad |(\sqrt{3} - 3) \cdot (\sqrt{3} - 1)| = \dots\dots\dots$$

2. Stabilisci se la seguente disuguaglianza è vera o falsa:

$$|x - 2| = \begin{cases} x - 2 & \text{se } x \geq 0 \\ 2 - x & \text{se } x < 0 \end{cases}$$

3. Scrivi a cosa è uguale $|x^2 - x - 2|$ in base alla definizione di valore assoluto.

4. Giustifica perché $|x^2 - x + 1| = x^2 - x + 1 \quad \forall x \in \mathbb{R}$

5. Completa in base alla definizione:

$$\text{esempio: } |x - 1| = \begin{cases} x - 1 & \text{se } x \geq 1 \\ 1 - x & \text{se } x < 1 \end{cases}$$

a) $|2x| =$

b) $|3 - x| =$

c) $|x^2 - 1| =$

d) $|x^2 - 3x| =$

e) $\left| \frac{x}{x - 4} \right| =$

f) $|-x^2 + 4x - 5| =$

Risolvi le seguenti equazioni:

6. $|x^2 - 3x| = -3$

$$S = \{\emptyset\}$$

$$7. \quad |x^2 - 2x| = 0 \quad S = \{0; 2\}$$

$$8. \quad |x^2 - 2x| = 3 \quad S = \{-1; 3\}$$

$$9. \quad |x + 5| - 4 = 1 \quad S = \{-10; 0\}$$

$$10. \quad |2x - 3| = 4 \quad S = \left\{-\frac{1}{2}; \frac{7}{2}\right\}$$

$$11. \quad |x^2 + 2x| = 2 \quad S = \{-1 \pm \sqrt{3}\}$$

$$12. \quad |x^2 - 3x + 2| = 0 \quad S = \{1; 2\}$$

$$13. \quad |x^3 + x^2 + x| = -5 \quad S = \{\emptyset\}$$

$$14. \quad \left|\frac{x-2}{3}\right| = \frac{1}{2} \quad S = \left\{\frac{1}{2}; \frac{7}{2}\right\}$$

$$15. \quad |2 - x^2 + x| = 2 \quad S = \left\{0; 1; \frac{1 \pm \sqrt{17}}{2}\right\}$$

$$16. \quad \left|\frac{x-4}{x+1}\right| = 3 \quad S = \left\{-\frac{7}{2}; \frac{1}{4}\right\}$$

$$17. \quad \left|\frac{1-3x}{2+5x}\right| = 0 \quad S = \left\{\frac{1}{3}\right\}$$

$$18. \quad \left|\frac{x+5}{2x+6}\right| = -10 \quad S = \{\emptyset\}$$

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$$19. \quad |2x + 5| = x - 1 \quad S = \{\emptyset\}$$

$$20. \quad x - 3 = |4x + 1| \quad S = \{\emptyset\}$$

$$21. \quad |x + 2| = 3x + 1 \quad S = \left\{\frac{1}{2}\right\}$$

$$22. \quad |x^2 - 3x| = 2x \quad S = \{0; 1; 5\}$$

$$23. \quad \left| 1 - \frac{2}{x} \right| = x \quad S = \{1\}$$

$$24. \quad \left| \frac{x-1}{2} - \frac{x+2}{3} \right| = \frac{x}{6} - \frac{4}{3} \quad S = \{\emptyset\}$$

$$25. \quad |x+2| = 2x+3 \quad S = \{-1\}$$

$$26. \quad |x-1| = x^2 - x - 1 \quad S = \{-\sqrt{2}; 2\}$$

$$27. \quad |2x^2 - x| = x^2 + 2x \quad S = \{0; 3\}$$

$$28. \quad |x| = \frac{1}{2}(x+1) \quad S = \left\{-\frac{1}{3}; 1\right\}$$

$$29. \quad \left| x + \frac{1}{2}(x-1) \right| = \frac{x-2}{3} + 6 \quad S = \left\{-\frac{29}{11}; 5\right\}$$

$$30. \quad \frac{2x+2}{|x|} = \frac{4}{5} \quad S = \left\{-\frac{5}{7}\right\}$$

$$31. \quad \frac{|x+1|}{x} = 1 + \frac{2}{x} \quad S = \left\{-\frac{3}{2}\right\}$$

$$32. \quad \frac{|x|-9}{x^2-9} = 0 \quad S = \{\pm 9\}$$

$$33. \quad \frac{|x^2 - 2x + 3|}{x^2 - 1} = 1 \quad S = \{2\}$$

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$$34. \quad |x+1| = |3x-2| \quad S = \left\{\frac{1}{4}; \frac{3}{2}\right\}$$

$$35. \quad |2x-3| + |x| = 2 \quad S = \left\{1; \frac{5}{3}\right\}$$

$$36. \quad |x^2 - 1| = 3|x| \quad S = \left\{\frac{-3 \pm \sqrt{13}}{2}; \frac{3 \pm \sqrt{13}}{2}\right\}$$

$$37. \quad |x+2| = |3-2x| \quad S = \left\{\frac{1}{3}; 5\right\}$$

$$38. \quad |x| + |x+1| = 2 \qquad S = \left\{-\frac{3}{2}; \frac{1}{2}\right\}$$

$$39. \quad |x^2 + 5x| = |x^2 - 2x| \qquad S = \left\{-\frac{3}{2}; 0\right\}$$

$$40. \quad |1-x| + |x-3| = x^2 - x \qquad S = \left\{\frac{-1-\sqrt{17}}{2}; 2\right\}$$

$$41. \quad |x^2 - 1| = |x^2 - x + 1| \qquad S = \left\{0; \frac{1}{2}; 2\right\}$$

$$42. \quad |x-1| = -|x-2| \qquad S = \{\emptyset\}$$

$$43. \quad |x+2| + x = x|x| - 1 \qquad S = \{3\}$$

$$44. \quad |x^3| + |x| = 0 \qquad S = \{0\}$$

DISEQUAZIONI CON MODULO

Risolvi le seguenti disequazioni:

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$$45. \quad |2x+5| < -3 \qquad S = \{\nexists x \in R\}$$

$$46. \quad |x| + 2 < 9 \qquad S = \{-7 < x < 7\}$$

$$47. \quad |4x-5| + 4 > 5 \qquad S = \left\{x < 1 \cup x > \frac{3}{2}\right\}$$

$$48. \quad |x^3 - 9| > -6 \qquad S = \{\forall x \in R\}$$

$$49. \quad |4x-3| + 1 > 10 \qquad S = \left\{x < -\frac{3}{2} \cup x > 3\right\}$$

$$50. \quad 1 > 12 - |x| \qquad S = \{x < -11 \cup x > 11\}$$

$$51. \quad |4x^2 - 1| + 5 < 0 \qquad S = \{\nexists x \in R\}$$

$$52. \quad |1-2x| < 1 \qquad S = \{0 < x < 1\}$$

$$53. \quad |x^2 - 4| < 5 \qquad S = \{-3 < x < 3\}$$

$$54. \quad \left| \frac{x^2 - 1}{2x + 1} \right| < 1 \qquad S = \{-2 < x < 1 - \sqrt{3} \cup 0 < x < 1 + \sqrt{3}\}$$

$$55. \quad |3x| < 12 \qquad S = \{-4 < x < 4\}$$

$$56. \quad |x^2 - (x - 3)^2 + x| < 2 \qquad S = \left\{1 < x < \frac{11}{7}\right\}$$

$$57. \quad 4|x| - 9 > 3|x| + 3 \qquad S = \{x < -12 \cup x > 12\}$$

$$58. \quad -|x - 6| \geq 0 \qquad S = \{x = 6\}$$

$$59. \quad \left| \frac{x^2 - 2x}{x + 1} \right| < 0 \qquad S = \{\nexists x \in R\}$$

$$60. \quad \left| \frac{x - 3}{x + 2} \right| > 0 \qquad S = \{x \neq -2 \cup x \neq 3\}$$

$$61. \quad \left| \frac{2 - 5x}{2x + 3} \right| \leq 7 \qquad S = \left\{x \leq -\frac{23}{9} \cup x \geq -1\right\}$$

$$62. \quad |4x^2 - 16| \leq -5 \qquad S = \{\nexists x \in R\}$$

$$63. \quad \left| \frac{x - 2}{x + 3} \right| \geq 4 \qquad S = \left\{-\frac{14}{3} \leq x < -3 \cup -3 < x \leq -2\right\}$$

$$64. \quad |4x - 3| + 2 \geq 0 \qquad S = \{\forall x \in R\}$$

$$65. \quad \left| \frac{2x - 1}{x - 4} \right| \geq 0 \qquad S = \{x \neq 4\}$$

$$66. \quad |x^2 + 2x| \leq 3 \qquad S = \{-3 \leq x \leq 1\}$$

$$67. \quad 6 - \left| \frac{x - 1}{x + 1} \right| \leq 0 \qquad S = \left\{-\frac{7}{5} \leq x < -1 \cup -1 < x \leq -\frac{5}{7}\right\}$$

$$68. \quad 2 - \left| \frac{x}{2x+1} \right| \geq 0$$

$$S = \left\{ x \leq -\frac{2}{3} \cup x \geq -\frac{2}{5} \right\}$$

$$69. \quad |x^2 - 9| \leq -3$$

$$S = \{ \nexists x \in R \}$$

$$70. \quad \frac{2}{|3x-5|} \geq 1$$

$$S = \left\{ 1 \leq x < \frac{5}{3} \cup \frac{5}{3} < x \leq \frac{7}{3} \right\}$$

$$71. \quad 3 + |x-2| > 8 - |x-2|$$

$$S = \left\{ x < -\frac{1}{2} \cup x > \frac{9}{2} \right\}$$

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$$72. \quad |3x-4| \geq 2x+5$$

$$S = \left\{ x \leq -\frac{1}{5} \cup x \geq 9 \right\}$$

$$73. \quad |x| > x^2 - 4x + 6$$

$$S = \{ 2 < x < 3 \}$$

$$74. \quad |5-2x| < x-3$$

$$S = \{ \nexists x \in R \}$$

$$75. \quad |x^2 - 4| < x^2 + 5x + 10$$

$$S = \left\{ x > -\frac{14}{5} \right\}$$

$$76. \quad |4x-3| \leq 2x+5$$

$$S = \left\{ -\frac{1}{3} \leq x \leq 4 \right\}$$

$$77. \quad |-2x+5| < x-3$$

$$S = \{ \nexists x \in R \}$$

$$78. \quad |x^2 - 4| \geq 4 + 2x$$

$$S = \{ x \leq 0 \cup x \geq 4 \}$$

$$79. \quad 2x + 3|x| - 4 \geq 0$$

$$S = \left\{ x \leq -4 \cup x \geq \frac{4}{5} \right\}$$

$$80. \quad x - |x+1| \geq 4$$

$$S = \{ \nexists x \in R \}$$

$$81. \quad \frac{|x+1|}{x} - 1 < \frac{3}{x}$$

$$S = \{ x < -2 \cup x > 0 \}$$

$$82. \quad |x^2 - 4| > 4x - 8$$

$$S = \{ x \neq 2 \}$$

$$83. \quad \frac{1}{|x+1|} > 1$$

$$S = \{-2 < x < -1 \cup -1 < x < 0\}$$

$$84. \quad (1 - |x|)^2 - 2x - 3 \leq 0$$

$$S = \{-\sqrt{2} \leq x \leq 2 + \sqrt{6}\}$$

$$85. \quad \frac{|4x-1| - 3x - 1}{x^2 + 4x + 16} > 0$$

$$S = \{x < 0 \cup x > 2\}$$

$$86. \quad 2 > x - |x+2|$$

$$S = \{\forall x \in \mathbb{R}\}$$

$$87. \quad 2x - 5 < |x+2|$$

$$S = \{x < 7\}$$

$$88. \quad 3x + 4 \geq |3x+2|$$

$$S = \{x \geq -1\}$$

$$89. \quad 2x^2 + 2x \leq |2x^2 - 12x|$$

$$S = \left\{x \leq \frac{5}{2}\right\}$$

$$90. \quad x + 2 > \frac{|4 - x^2|}{2}$$

$$S = \{0 < x < 4\}$$

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$$91. \quad 2x - |-x| \leq |x-2|$$

$$S = \{x \leq 1\}$$

$$92. \quad |x| + 6x > 2|3x-6|$$

$$S = \left\{x > \frac{12}{13}\right\}$$

$$93. \quad |x+1| - 5 < |3-2x|$$

$$S = \{\forall x \in \mathbb{R}\}$$

$$94. \quad |3x-1| \leq 2x - |x-3|$$

$$S = \{\nexists x \in \mathbb{R}\}$$

$$95. \quad \frac{|2x-3| - 1}{|x| - 2} \leq 0$$

$$S = \{-2 < x \leq 1\}$$

$$96. \quad \frac{-|x|}{|x-1|} \geq 0$$

$$S = \{x = 0\}$$